# Results of a Study to Determine the Probability of Chance Match Occurrences Between Fibers Known to be from Different Sources 

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#### Abstract

The results of a study to determine the frequency of chance match occurrences among fibers known to be from different sources is described. Fibers from articles of clothing of individuals from cases were recovered through an adhesive rolling device and segregated to microscope slides under Cargille H. D. refractive index liquid $N_{D}^{25}=1.525$. They were identified as to type through polarizing microscopy and dispersion staining techniques, and characterized through morphological characteristics and color. The information developed from these steps was entered onto a multi-sort card to permit retrieval of fibers for microscopical comparison by groups. Effectively, 283882 comparisons involving control and random fibers from 40 articles of fabric were performed. The results reported are believed to give realistic estimates of the probability of chance fiber matches as they can occur in practice. Also given are methods for computing probability, and a discussion of the results.


KEYWORDS; criminalistics, fibers, microscopy, probability

Fibers are ubiquitous. It is knowledge of this fact by everyone that prevents their ready acceptance as significant physical evidence when they are presented as comparison matches ${ }^{2}$ by the forensic criminalist. This is a problem that assumes critical proportions when the fiber matches must stand virtually alone as the principal evidence against the defendant. In just such a case, after having been shown a series of fiber matches effected between articles associated with the victim and suspect, the prosecuting attorney raised his hands in a plaintive gesture of appeal as he asked, "What does it all mean?"

Indeed, "What does it all mean?" The author has dealt with this question through use of reasoned explanations based upon experience and intuition. Such explanations have been lacking in probability estimates of chance occurrence based upon personally obtained quantitative data.

Other workers have addressed this problem. Burd and Kirk [1] have described a study of the incidence of matching blue wool fibers found in men's suit materials. More recently, Cook and Jackson [2] have described their investigations to determine the incidence of finding common fibers on clothing and car seats.

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${ }^{1}$ Criminalists, Office of the District Attorney, Santa Clara County Laboratory of Criminalistics, San Jose, CA.
${ }^{2}$ The term match and its analogues as they are used in the context of this article are used in the sense that one fiber is the microscopic counterpart of another in all optically determined properties.

The designs of these studies have a characteristic in common: a target fiber or set of target fibers, selected as being common, is specified and then searched for in garments sampled from a parent population. The rationale of these studies is that since the target fibers are relatively common, match occurrences involving these fibers could be expected to be relatively frequent, and computations of probabilities based upon these occurrences should give results correspondingly conservative. These studies have provided useful information. They have the advantage of reducing the amount of work required for comprehensive studies involving a wider range of fibers. However, the most realistic answers are those provided by a study designed to show the frequency of chance match occurrences from fibers as they are encountered in practice. Such encounters surely include randomly adherent fibers as well as control target fibers. The authors believe that this is the study design of choice. It is the purpose of this article to describe the results of such a study.

## Experimental Procedures

The study fibers were obtained from articles of clothing entering the Laboratory of Criminalistics, Santa Clara County, California during the period 1981 to 1984. A wide variety of cases was represented, but the principal sources were cases involving crimes against persons. One article from each case was used. When a case included many articles of clothing, the one used was selected in a manner to insure that all had an equal chance of inclusion. Exceptions were made when an article happened to be heavily bloodstained.

Procedures previously described [3] were used for recovering adherent fibers, segregating to locations on microscope slides, fixing under a coverslip in Cargille H.D. refracture index liquid $N_{D}^{25}=1.525$, and numbering on the coverslip for identification. Identification of fiber types was effected through a scheme of identification ${ }^{3}$ also previously described [4]. The method was supplemented as occasionally required by use of a Berek compensator inserted into the optical path of the microscope between crossed polars. Carroll [5] has recently brought deservedly renewed attention to the value of this optical device for identifying synthetic fibers.

Informational data characterizing each fiber were entered by code designation onto a multipurpose deck Indecks ${ }^{(\widehat{s})}$ card. The source of the fiber, by case number and item designation, together with the slide and fiber number designation, were handwritten on the face of the card; also, a hand drawn sketch was made. A log was maintained to provide a source-to-source accounting of the number of fibers characterized. This was done to permit the accurate computation of the real number of intercomparisons required. At the completion of this phase of the study, 763 fibers from 40 sources had been characterized giving an average of 19 distinguishable fibers by type, color, and other characteristics from each source. Tables 1 and 2 give frequency of occurrence data that emerged from these characterizations.

Fibers qualifying for direct comparison by groups were identified through card sort of the multipurpose index cards. The slides identified were retrieved from storage and the required fibers for comparison located on the proper slide through the fiber number assignment. The number of comparisons required in a group ranged from 1 (group of 2 sources) to 276 (group of

[^0]TABLE 1-Incidence of fibers by type and color determined from 763 control and random fibers found on fabric articles from 40 different sources; also shown is the incidence of match occurrences by fiber type and color.

| Color ${ }^{\text {b }}$ | Fiber Type ${ }^{a}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ACE | TAC | ACR | OFN | DYL | VRL | RYN | PAM | PYE | COT | WLN | AML | UND | Total | \% of Total |
| Purple | $\ldots$ | $\ldots$ | $\ldots$ | 1 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 3 | 4 | $\ldots$ | $\ldots$ | 8 | 1.0 |
| Blue-purple | 1 | 1 | 6 | ... | $\ldots$ | 3 | $\ldots$ | $\cdots$ | $\ldots$ | 3 | 11 |  | 1 | 26 | 3.4 |
| Blue | 1 | 2 | 27 | $\ldots$ | $\ldots$ | 4 | $\ldots$ | 11 | 36 | 59 | 35 | 1 | 7 | 183 | 24.0 |
|  |  |  |  |  |  |  |  |  | 1M | 11 M | 1M |  |  |  |  |
| Blue-green | $\cdots$ | $\ldots$ | 7 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 2 | 4 | 7 | 7 | 1 | $\ldots$ | 28 | 3.7 |
| Green | 1 | $\ldots$ | 15 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 2 | 5 | 6 | 8 | ... | $\ldots$ | 37 | 4.8 |
| Yellow-green | $\ldots$ | $\ldots$ | 1 | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | 3 | 1 | 1 | $\ldots$ | 1 | 7 | 0.9 |
| Yellow | 1 | $\ldots$ | 5 | $\ldots$ | $\ldots$ | $\ldots$ | . | 7 | 2 | 3 | 3 | $\ldots$ | 1 | 22 | 2.9 |
| Yellow-orange ${ }^{\text {c }}$ | 5 | $\ldots$ | 22 | $\ldots$ | $\ldots$ | $\ldots$ | 2 | 4 | 3 | 13 | 6 | 1 | 4 | 60 | 7.9 |
|  |  |  | $1 \mathrm{M}^{e}$ |  |  |  |  |  |  |  |  |  |  |  |  |
| Orange ${ }^{c}$ | $\ldots$ | $\ldots$ | 13 | $\ldots$ | $\ldots$ | $\ldots$ |  | 7 | 6 | 10 | 5 | $\ldots$ | 3 | 44 | 5.8 |
| Red-orange ${ }^{c}$ | $\ldots$ | $\ldots$ | 17 | $\ldots$ | $\cdots$ | 1 | 2 | 4 | 2 | 3 | 4 | $\ldots$ | $\ldots$ | 33 | 4.3 |
| Red | 5 | 6 | 62 | $\ldots$ | 1 | 3 | 2 | 1 | 24 | 28 | 25 | 4 | 3 | 164 | 21.5 |
|  |  |  |  |  |  |  |  |  |  | 2M |  |  |  |  |  |
| Purple-red | $\cdots$ | $\ldots$ | 5 | 1 | $\cdots$ | 1 | $\ldots$ | 1 | 5 | 4 | 2 | 1 | 1 | 21 | 2.7 |
| Colorless or near | 2 | $\ldots$ | 7 | 1 | $\cdots$ | $\cdots$ | 2 | 2 | 14 | 9 | 4 | 4 | 3 | 48 | 6.3 |
|  |  |  |  |  |  |  |  |  | 3M |  |  |  |  |  |  |
| Gray to black | $\ldots$ | $\ldots$ | 11 | $\ldots$ | 3 | 1 | $\ldots$ | 1 | 2 | 6 | 3 | 2 | 10 | 39 | 5.1 |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 1M |  |  |
| Cross-range ${ }^{\text {d }}$ | 1 | $\cdots$ | 6 | $\cdots$ | $\ldots$ | 3 | $\cdots$ | 7 | 4 | 19 | 2 | 1 | 1 | 44 | 5.8 |
| Total | 17 | 9 | 204 | 3 | 4 | 15 | 8 | 49 | 110 | 174 | 120 | 15 | 35 | 763 | 100.0 |
| \% of Total | 2.2 | 1.2 | 26.7 | 0.4 | 0.5 | 1.96 | 1.04 | 6.4 | 14.4 | 22.8 | 15.8 | 1.96 | 4.6 | ... | 100.0 |

${ }^{a}$ Fiber type abbreviations: $\mathrm{ACE}=$ acetate rayon, $\mathrm{TAC}=$ triacetate rayon, $\mathrm{ACR}=$ acrylic, $\mathrm{OFN}=$ olefin, $\mathrm{DYL}=\mathrm{Dyne}{ }^{\oplus}, \mathrm{VRL}=\mathrm{Vere}{ }^{\oplus}, \mathrm{RYN}=$ rayon, PAM $=$ polyamide, $\mathrm{PYE}=$ polyester, $\mathrm{COT}=$ cotton, $\mathrm{WLN}=$ wool, $\mathrm{AML}=$ animal fibers other than wool (usually rabbit wool hair), and UND $=$ unidentified. ${ }^{b}$ Colors are given as ranges; each range includes six class steps.
${ }^{c}$ Includes browns.
${ }^{e}$ The letter $M$ follows the number of match occurrences found at the appropriate type fiber and color location.

TABLE 2-Incidence of microscopically observed morphological characteristics among 419 man-made fibers ${ }^{a}$ found on fabric articles from 40 different sources.

| Morphological Characteristic | Fiber Type ${ }^{\text {b }}$ |  |  |  |  |  |  |  |  |  | $\%$ of Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ACE | TAC | ACR | OFN | DYL | VRL | RYN | PAM | PYE | Total |  |
| Plain or striated | 15 | 5 | 73 | 3 | 4 | 8 | 8 | 11 | 48 | 175 | 41.9 |
| Lobated | $\cdots$ | $\cdots$ | 8 | ... | ... |  | ... | 26 | 3 | 37 | 8.8 |
| Delustered | 2 | 4 | 123 | $\ldots$ | $\ldots$ | 7 | $\ldots$ | 9 | 57 | 202 | 48.2 |
| Lobated and delustered | $\cdots$ | $\cdots$ | $\cdots$ | $\ldots$ | ... | $\cdots$ | $\ldots$ | 3 | 2 | $\frac{5}{419}$ | 1.2 |

${ }^{a}$ Does not include unidentified fibers.
${ }^{b}$ See Table 1 for explanation of abbreviations for column headings.

24 sources). Direct comparison of the fibers was performed by slide interchange on a microscope and viewed at $\times 100$ magnification. A preliminary comparison under a Greenough binocular stereoscopic microcope at $\times 9$ to $\times 27$ magnification was performed when the number of intercomparisons in a group was large. This was done by superimposing of the mounted fibers, one over the other, and focussing up and down. Eliminations at this stage based upon clear differences reduced the number of comparisons required at higher magnification. Table 1 gives the match occurrences, indicated by the letter $M$, resulting by fiber type and color. Table 3 summarizes the match occurrences by fiber type and gives the calculated probabilities by source.

## Computation of Probabilities

The probability of one, two, three, four, and so forth match occurrences can be calculated from Poisson's probability formula

$$
P_{m}=\frac{\lambda^{m}}{m!} \cdot e^{-\lambda}
$$

TABLE 3-The number of match occurrences and the probability of chance match occurrences determined from the intercomparison of 763 fibers from 40 different sources given by probability class."

| Probability Class | Match Occurrences Found | Calculated Probability <br> by Source |
| :--- | :---: | :---: |
| 1. Fiber type |  |  |
| Incidence less <br> than $10 \%^{b}$ | 0 |  |
| 2. All fibers less |  |  |
| blue cotton | $6^{c}$ | $1 / 1000$ |
| 3. All fibers | $17^{c}$ | $1 / 130$ |

[^1]where
$e=2.7183$ (base of Napierian logarithms);
$\lambda=n \times p$, where $n$ is the number of trials, that is, comparisons, and $p$ is the probability of the match occurrence; and
$m=$ match occurrence(s).
Note that the value of $e^{-\lambda}$ is the probability of a match not occurring. The use of this formula is illustrated by examples. For convenience, the examples use $1 / 100$ for the probability of a match occurrence and 20 for the average number of distinguished fibers for each source in place of the values found in this study, respectively, 1/130 and 19.

## Example 1

Source 1 yields 20 distinguishable fibers by type and color as does Source 2. Microscopical comparison examinations reveal one Class 2 probability match as defined in Table 3. What is the probability of this chance match occurrence? The required probability $P_{1}$ is given by Poisson's formula

$$
P_{1}=0.01 / 1 \times 0.99=1 / 100
$$

where $\lambda=n \times p, n=20 / 20 \times 20 / 20, p=1 / 100$, and $m=1$.

## Example 2

The same conditions as given in Example 1 are used except that Source 2 involves only a single distinguishable fiber. The probability $P_{1}$ is given by

$$
P_{1}=0.0005 / 1 \times 0.9995=1 / 2001
$$

where $n=20 / 20 \times 1 / 20, p=1 / 100$, and $m=1$.

## Example 3

The same conditions as given in Example 1 are used except that both sources yield one distinguishable fiber. The probability $P_{1}$ is given by

$$
\begin{aligned}
& \qquad \begin{aligned}
P_{1} & =0.000025 / 1 \times 0.999975=1 / 40001 \\
\text { where } n=1 / 20 \times 1 / 20, p & =1 / 100, \text { and } m=1
\end{aligned}
\end{aligned}
$$

## Example 4

The same conditions are used as given in Example 1 except that three matches are found. The probability $P_{3}$ is given by

$$
P_{3}=0.000001 / 6 \times 0.999999=1 / 6000000
$$

where $n=20 / 20 \times 20 / 20, p=1 / 100$, and $m=3$.

## Example 5

Source 1 yields 20 distinguishable fibers while Source 2 yields 40 distinguishable fibers. Two matches, both involving probability Class 1 fibers, are found. What is the probability of chance occurrence of these matches? The probability $P_{2}$ is given by

$$
P_{2}=0.0022 / 2 \times 0.998=1 / 501002
$$

where $n=20 / 20 \times 40 / 20, p=1 / 1000$, and $m=2$.
The validity of the application of Poisson's probability formula (1837) to study results in which the number of occurrences can be stated, but not the number of nonoccurrences, is discussed by Langley [6]. The mathematical proof of its approximation to the binomial approach, as used by Gaudette and Keeping [7] in their article in a similar study involving hair, is given by Wilks [8].

Nylon, a type of fiber yielding relatively frequent match occurrences in case experience, was not represented in any of the matches found. Its frequency of occurrence, $6.4 \%$, as a type of fiber given in Table 1 is also lower than expected. The explanation lies in the sources compared; in this study, they were garment versus garment contrasted with garmet versus carpet or other sources, for example, fabric covered furniture, experienced in case work.

An interesting and informative computation would be to find the number of comparisons by source ( $n$ ) sufficient to assure a probability of $>1 / 2$ of obtaining a match $m$ at least once when the probability of match is say $1 / 1000$. Such a computation is appropriate for nylon as well as other fiber types among which no matches resulted. This computation is given by

$$
n>\frac{\log 2}{\log 1000-\log 999}=692.8
$$

The number calculated is less than the number of intercomparisons by source in this study, that is, 780 , so that the probability of match, $1 / 1000$, is conservative. Further calculations would reveal $1 / 1127$ to be the probability of match of say 2 nylon fibers under the conditions given. The computation is the same as that which is applied to find the number of throws of two dice to assure a probability of $>1 / 2$ of obtaining double six at least once. This is done by solving the inequality

$$
\frac{35}{36}^{n}<1 / 2
$$

for $n$, we find

$$
n>\frac{\log 2}{\log 36-\log 35}=24.6
$$

The meaning is that in 25 throws there is more of a likelihood to obtain double six at least once than not to obtain it at all. The historical basis of this computation has been described by Uspensky [9].

## Discussion

When $m$ is several, say 3 to 5 , and $p$ is $1 / 1000, P m$ rapidly approaches infinetesimally small values. The validity of computations assigning equal $p$ for each subsequent match occurrence is subject to criticism on the grounds of dependency, that is, since one match occurrence has
been found, this supplementary knowledge affects the probability of showing additional matches. The logic of this contention is obscured by the contradiction that if dependency is admitted then the proof that the two articles were in contact is also effectively admitted. This must necessarily be the case in the absence of another explanation for the dependency. Whether the multiple fiber matches occurred by change, or by dependency is, in fact, the ultimate problem of proof.

If computations are performed, what limit for smallness of probability is to be set as a practical impossibility? The answer depends on the risk the worker is willing to assume, if contrary to expectation, chance occurrence with a small probability, for example, 1/1000, 1/100 000, $1 / 1000000$, should occur.

It has been observed from case experience that the incidence of adherent polyester fibers is lower than expected in consideration of their known widespread use as wash and wear clothing fabric. Table 1 confirms this observation by showing that polyesters comprise only $14.4 \%$ of the fibers distinguished, approximately equal to wool which was $15.8 \%$. The explanation lies in the mechanical properties of polyester fibers, for example, high strength, toughness, stiffness, and their frequent usage in tightly constructed fabrics reducing the tendency to pill [10]. The effect of these combined realities are accounted for in the practical design and performance of this study. Salter et al [11] have recently described the results of studies related to this observation.

In the execution of this study, a requirement applied was to consider that all fibers from any one source had an equal chance of matching all fibers in the 39 other sources. Under this consideration, the results of the study show a probability of a chance match occurrence of $1 / 41$. By omitting two matches involving colorless, delustered polyesters known to be control fibers, the probability becomes $1 / 46$. Discomforting, but not surprising, is the finding that 11 of the 17 re maining matches involved blue cottons of various shades. The cautious worker, under the realization of the known frequency of blue cotton fibers, would be justified in omitting them from consideration as evidence having significant value. This is especially true if differentiations by methods beyond microscopical comparisons are not performed. Omitting blue cotton matches gives a final probability of $\mathbf{1 / 1 3 0}$. However, as previously discussed under the computations section, it is reasonable to use a lower probability, for example, $1 / 1000$, when the fiber type is relatively infrequent, say less than $10 \%$. Also, it is mathematically correct to compound the two probabilities as the situation requires.

In addition to providing a value of $p$, probability of chance match occurrence, this study provides the value of $n$, the number of trials-comparisons based upon the average number of distinguishable fibers by source (Table 3). Both values must be known to effect computations, and, accordingly, the worker must record the number of distinguishable fibers by source when working a case. This is apparent from the illustrative examples given.

The frequency of match occurrences given are those revealed by microscopical comparisons. One match involved black fibers which resisted identification as to type by the methods used. In an actual case, other available methods would have been applied. By applying microscopical methods alone, the amount of work required to complete this study was reduced. It should be clear that the application of other methods for comparison differentiation, for example, thinlayer chromatography, microspectrophotometry, and so forth would have served only to reduce the probabilities given.

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The authors acknowledge the influence of the teachings of the late Dr. Paul L. Kirk, Professor of Criminalistics, University of California, Berkeley, CA. Dr. Kirk taught the practical benefits to one's understanding of physical evidence to be derived from personal study of what he called "special problems" as they arise in actual case work. Also, he encouraged the writing
of articles to report the results of these studies despite the always present risk of courting criticism. This teaching and encouragement have been applied in the execution of this study and the preparation of this article.

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[^0]:    ${ }^{3}$ Since the publication of this article additional experience has been gained and is passed on for the benefit of the reader:

    1. Olefin (polypropylene) fibers were encountered in a rape case. The control fibers were from a carpet on the rear deck of a station wagon. Central Stop dispersion staining colors in Cargille H.D. refractive index liquid ( $N_{D}^{25}=1.525$ ) and typical refractive indices for these fibers are: $N_{11}=$ blue or blue-magenta, 1.530, $N_{1}$ rainbow, 1.496 .
    2. Acetate fibers are dissolved, while triacetate fibers are visibly affected after approximately two to three years immersion in Cargille H.D. refractive index liquid ( $N_{D}^{25}=1.525$ ). It is recommended that matching fibers be recovered from their original mounts, washed with solvent, for example, xylene, and remounted either dry or in any of several resin based mounts-Permount ${ }^{\oplus}$, Canada Balsam, or XAM Improved Neutral White, to gain the advantages of permanency.
[^1]:    ${ }^{a}$ Average number of fibers from each source $=19.1$, standard deviation $=6.4, \mathrm{Ef}-$ fective number of fiber comparisons performed corrected for source $=283882$.
    ${ }^{b}$ See Table 1 for fiber types having incidence less than $10 \%$.
    ${ }^{c}$ Two matches involving colorless, delustered polyester control fibers not included.

